Summary 4: Recording Motion

1. Introduction

In the laboratory, motion can be investigated using a puck on an air table like the one shown in figure 1. As the puck travels on the table it leaves dot marks on the registration paper below it every fixed time interval called tau τ, that is controlled by a timer. It is clear that the farther apart the dots on the registration paper (or dot print), the faster the puck is moving.

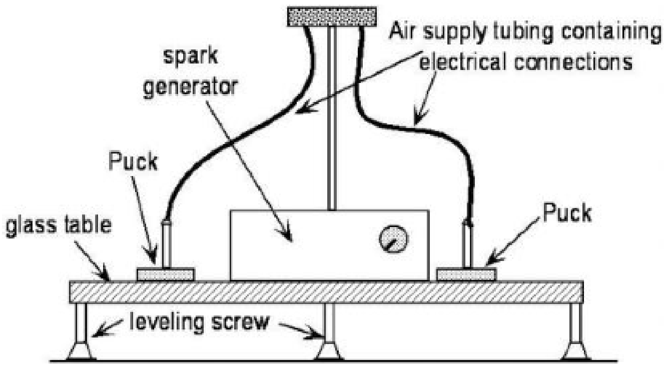

Figure 1. Air table

When we take the registration paper to study the motion, we mark the dots by A_0, A_1, A_2, A_3, *etc*... (at instants t_0, t_1, t_2, t_3, ... *respectively*). The time interval between each two successive points is τ, which is usually in milliseconds (one of the following values: 20ms, 40ms, 60ms, 80ms, 100ms). We measure the distance between each two successive points (A_0A_1, A_1A_2, A_2A_3, *etc*...).

2. Calculating average speed

The average speed between two instants $t_{initial}$ and t_{final} is

$$v_{av} = \frac{distance\ between\ A_{initial}\ and\ A_{final}}{time\ interval\ between\ t_{initial}\ and\ t_{final}} \quad (1)$$

3. Calculating instantaneous speed

The instantaneous speed v, at an instant t_k, is approximately equal to the average speed between two instants t_{k-1} and t_{k+1}. For example, the instantaneous speed at a point A_2 is

$$v_2 = \frac{A_1A_3}{t_3 - t_1} = \frac{A_1A_3}{2\tau}$$

So:

$$v_1 = \frac{A_0A_2}{2\tau} \tag{2}$$

$$v_2 = \frac{A_1A_3}{2\tau} \tag{3}$$

$$v_3 = \frac{A_2A_4}{2\tau} \tag{4}$$

$$v_4 = \frac{A_3A_5}{2\tau} \tag{5}$$

And so on...

Example 1: The following measurements are made on a registration paper of figure 2: $A_0A_1 = 2cm$, $A_1A_2 = 2.5cm$, $A_2A_3 = 4cm$, $A_3A_4 = 4.7cm$, $A_4A_5 = 5cm$. The time interval between each two successive points is $\tau = 20ms = 0.02s$.

Figure 2. Registration paper or dot print

The total time from A_0 to A_5 is five times τ. The average speed between t_0 and $t_5$is

$$v_{av} = \frac{A_0A_5}{5\tau} = \frac{2+2.5+4+4.7+5}{5\times 0.02} = \frac{18.2}{0.1} = 182cm/s$$

The instantaneous speeds at t_0, t_2, t_3, t_4 are respectively:

$$v_1 = \frac{A_0A_2}{2\tau} = \frac{2+2.5}{2\times 0.02} = 112.5cm/s$$

$$v_2 = \frac{A_1A_3}{2\tau} = \frac{2.5+4}{2\times 0.02} = 162.5cm/s$$

$$v_3 = \frac{A_2A_4}{2\tau} = \frac{4+4.7}{2\times 0.02} = 217.5cm/s$$

$$v_4 = \frac{A_3A_5}{2\tau} = \frac{4.7+5}{2\times 0.02} = 242.5cm/s$$

4. Calculating average acceleration

The average acceleration between two instants $t_{initial}$ and t_{final} is

$$a_{av} = \frac{\Delta v}{\Delta t} = \frac{v_{final} - v_{initial}}{\Delta t} \tag{6}$$

For example, the average acceleration in example 1 between t_1 and t_4 is (the total time from t_1 to t_4 is three times τ):

$$a_{av} = \frac{v_4 - v_1}{3\tau} = \frac{242.5 - 112.5}{3 \times 0.02} = 2166.7 cm/s^2$$

5. Calculating instantaneous acceleration

The instantaneous acceleration a, at an instant t_k, is approximately equal to the average acceleration between two instants t_{k-1} and t_{k+1}. For example, the instantaneous acceleration at the instant t_2 is

$$a_2 = \frac{\Delta v}{\Delta t} = \frac{v_3 - v_1}{t_3 - t_1} = \frac{v_3 - v_1}{2\tau} \tag{7}$$

So:

$$a_2 = \frac{v_3 - v_1}{2\tau} \tag{8}$$

$$a_3 = \frac{v_4 - v_2}{2\tau} \tag{9}$$

And so on...

For example, the instantaneous accelerations at t_2 *and* t_3 are respectively

$$a_2 = \frac{v_3 - v_1}{2\tau} = \frac{217.5 - 112.5}{2 \times 0.02} = 2625 cm/s^2$$

$$a_3 = \frac{v_4 - v_2}{2\tau} = \frac{242.5 - 162.5}{2 \times 0.02} = 2000 cm/s^2$$

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