Summary 3: Equations of Motion

1. Uniform Rectilinear Motion (URM)

When the trajectory of the moving object is straight line, and its speed is constant, we say its motion is uniform rectilinear motion (URM).

1.1. Definition: If we study the motion of an object on the x-axis, then the following symbols are used:

t_0: initial instant of time (the instant at which we start the timer which is taken as 0).
t: final instant of time (the instant at which we stop the timer).
v_0: initial velocity at instant $t_0 = 0s$.
v: final velocity at instant t.
x_0: initial abscissa at instant $t_0 = 0s$.
x: final abscissa at instant t.
$\Delta x = x - x_0$: displacement.
a: acceleration.

1.2. Equation of motion of URM

The only equation of motion of URM is

$$x = vt + x_0 \tag{1}$$

Example 1: A particle moves with constant speed of 20m/s on the x-axis in the positive sense starting from the origin. Find its abscissa 15s later.
Answer:
Started from origin $\Rightarrow x_0 = 0$
Moving in the positive sense$\Rightarrow v = +20m/s$.

$$x = vt + x_0 = 20 \times 15 + 0 = 300m$$

Example 2: Find the time needed for a car moving with velocity -5m/s starting from position of abscissa -3m to reach the position of abscissa -300m.
Answer:

$$\begin{aligned} x &= vt + x_0 \\ -300 &= -5t + (-3) \\ -300 + 3 &= -5t \\ t &= \frac{297}{5} = 59.4s \end{aligned}$$

2. Uniformly Varied Rectilinear Motion (UVRM)

When the trajectory of the moving object is straight line and its acceleration is constant (which means that the speed is uniformly varying), then we say that its motion is uniformly varied rectilinear motion (UVRM).

2.1. Equations of motion of UVRM

There are four equations of motion of UVRM, using five symbols: Δx, v_0, v, a, t.

$$v = at + v_0 \tag{2}$$

$$\Delta x = \frac{1}{2}at^2 + v_0 t \tag{3}$$

$$\Delta x = \frac{1}{2}(v_0 + v)t \tag{4}$$

$$v^2 - v_0^2 = 2a\Delta x \tag{5}$$

Each equation links four symbols out of five. If three of the quantities Δx, v_0, v, a, t are known, you can calculate the value of the fourth using the appropriate equation. Be careful that Δx, v_0, v, a, t are algebraic quantities, so you must allow for their signs when substituting numbers in the equations. For example, if moving in the negative direction the velocity and displacement must be negative. Note also that a deceleration is a negative acceleration.

Example 3: A car accelerates from rest at $5m/s^2$ along a straight road. How far has the car travelled after 3s?
Answer: In this case,
Δx is the quantity to be found
$v_0 = 0$, because the car started from rest.
$a = 5m/s^2$
$t = 4s$
So, choosing the equation which includes Δx, v_0, a *and* t but not v:

$$\Delta x = \frac{1}{2}at^2 + v_0 t = \frac{1}{2} \times 5 \times 3^2 + 0 \times 3 = 22.5m$$

Example 4: A car is travelling at 25m/s along a straight road. The driver puts the brakes on for 5s. If this causes a deceleration of $3m/s^2$, what is the car's final velocity?
Answer: In this case,
$v_0 = 25m/s$ (taking the positive sense of the x-axis the same as the sense of motion of the car)
v is the quantity to be found
$a = -3m/s^2$
$t = 5s$
So choosing the equation which includes v_0, v, a, *and* t but not Δx:

$$v = at + v_0 = -3 \times 5 + 25 = 10m/s$$

Example 5: An electron starts from rest, travells a distance of 15cm with constant acceleration and hits a screen at a speed of $3 \times 10^5 m/s$. Calculate the acceleration of the electron.
Answer: First we transform the unit of displacement to base SI unit, 15cm=0.15m. In this case,
$\Delta x = 0.15m$
$v_0 = 0$, because the car started from rest
$v = 3 \times 10^6 m/s$
a is the quantity to be found
So, choosing the equation which includes Δx, v_0, a *and* v but not t:

$$v^2 - v_0^2 = 2a\Delta x$$
$$(3 \times 10^6)^2 - 0 = 2 \times a \times 0.15$$
$$9 \times 10^{12} = 0.3 \times a$$
$$a = \frac{9 \times 10^{12}}{0.3}$$
$$a = 3 \times 10^{13} m/s^2$$

Farid Minawi's Physics Zone
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